Elderly People Working, Capital Accumulation, and Production Activities

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This paper examines the influence of elderly people working on capital accumulation and the level of production in the overlapping generations model. Elderly people working has a negative influence on capital accumulation and, under specific conditions, it has a negative influence on the production level.

Key words: Elderly people working, Overlapping generations model

Introduction
The population in advanced countries is rapidly aging. Aging of a country's population causes serious problems concerning financing pensions, care for elderly people, medical systems and so on. In addition, the employment of elderly people is an important issue. Therefore, it is important to examine the influence of elderly people working on the economy.

Employing the overlapping generations model, this paper investigates the influence of elderly people working on capital accumulation and the production level.

A comparison is made between the typical overlapping generations model where elderly people are not engaged in working and our original overlapping generations model where elderly people work.

There is little research in this area apart from that of Matsuyama (2002) and Futagami and Nakajima (2001).

Employing the same two-period overlapping generations model that is used in this study, Matsuyama (2002) analyzed a model in which workers can choose either retirement or continuing to work. His study reported the following two points:

(1) Workers work when they are old where their wages were low when they were younger.

(2) When workers continue to work when they are old, capital accumulation is low because they did not make sufficient savings when they were younger.

Our original model is different from Matsuyama (2002) in the following two points:

(1) In the model of Matsuyama (2002), analysis is made on the influences that elderly people working has on capital accumulation. However, in our original model analysis is made on the influences that elderly people working has on both capital accumulation and production level.

(2) Our model is based on the premise that all elderly people are expected to work. In contrast, in the model of Matsuyama (2002), elderly people can choose whether to continue to work or retire.

Concerning the second point, the reason for our premise is that we are concerned with the analysis of the situation where elderly people are normally engaged in work.

Employing a continuous time version of the overlapping generations model, Futagami and Nakajima (2001) report that extension of the retirement age (delayed retirement) has a negative influence on capital accumulation and economic growth.

Our model is different from that of Futagami and Nakajima (2001) in the following two points:

(1) The production function of our model is the Cobb-Douglas type, and the production function of the Futagami and Nakajima (2001) is the model type where the output-capital coefficient remains constant.

(2) Our model is a two-period overlapping generations model where one generation lives for two periods, but the model of Futagami and Nakajima (2001) is a continuous-time overlapping generations model where one generation lives for periods.

Juding from (1), the model of Futagami and Nakajima (2001) is clearly different from ours, and is unable to analyze how elderly people working affects the economy in terms of the change of output-capital coefficient. This difference leads to different conclusions about the influence on production level (economic growth) caused by elderly people working (retirement adjournment). Concerning (2), we chose our model in order to avoid complicated calculations. We will study the multi-period overlapping generations model in the Cobb-Douglas type in future.
1. Typical overlapping generation model

In this section, we review the typical overlapping generations model described in the standard textbook in order to compare it with our model discussed in the next section.

We use a logarithmic utility function in our review for simplification. In the typical overlapping generations model, people are supposed to adopt the following behaviors. They divide their wages \( (w') \) when they are younger (period t) into consumption \( (c') \) and saving \( (s') \). The superscripts of the variables indicate the period when the generation bears, and the subscripts indicate the time when the behavior is carried out. Therefore, \( c_{t+i} \) represents the consumption of period \( t+1 \) made by the generation bearing in \( t \) period. Savings including interest after retirement is spent when elderly (period \( t+1 \)).

From the behaviors mentioned above, the utility function and budget constraints are as follows:

\[
U = \log c' + \log \frac{c'_{t+i}}{1+\rho}
\]

\[
c' + s' = \omega_i
\]

\[
c'_{t+i} = (1+r_{t+i})s'
\]

\( \rho \) indicates time preference rate. From (1) to (3), the following equation is derived.

\[
U = \log(\omega - s') + \log \frac{1+r_{t+i}}{1+\rho}
\]

In order to find the level of optimal savings of a worker who bears in period \( t \) \((s')\), let \( \partial U / \partial c' = 0 \). Then, the next equation is derived.

\[
s'_{t+i} = \frac{\omega_i}{2+\rho}
\]

Next, we consider the behaviors of firms. These firms are supposed to be symmetrically monopolistic competitive firms. The duration period of their capital is assumed to be single. In this case, firm \( j \) tries to maximize its objective function \( \Theta \), shown in (6) at the time of investment decision.

\[
\Theta = P_i I' + \frac{p_{i+1} Y'_{i+1} - w_{i+1} L'_{i+1}}{1+i_{t+i}}
\]

\( Y, I, \) and \( L \) indicate production level, volume of investment, and volume of labor demand of the whole economy respectively. The variables with superscript \( j \) are the variables relating to the firm \( j \). That is, for example, \( I' \) means the volume of investment of firm \( j \).

\( P, w, \) and \( i \) represent prices, nominal wage rate, and nominal rate of interest respectively. The constraint conditions to maximize (6) are presented by the following equations:

\[
Y'_{i+1} = (K'_{i+1})^\gamma (L'_{i+1})^{1-\alpha}
0 < \alpha < 1
\]

(7)

\[
Y'_{i+1} = \left( \frac{P_{i+1}}{P_i} \right)^\theta Y'_{i+1}
\]

(8)

\[
K'_{i+1} = I'
\]

(9)

(7) shows the production function, which is of the Cobb-Douglas type in our models. (8) shows the individual demand curve\(^1\). Here, \( Y' \) means the nominal average demand to a firm. (9) represents capital accumulation when the duration period of capital is assumed to be single.

From (6) to (9), objective function \( \Theta \) is as follows:

\[
\Theta = P_i K'_{i+1} + \frac{\left( \frac{P_{i+1}}{P_i} \right)^\theta Y'_{i+1} - w_{i+1} L'_{i+1}}{1+i_{t+i}}
\]

(10)

In order to solve the profit-maximization problem, let \( \partial \Theta_i / \partial K'_{i+1} = 0 \). Then the following equation is derived.

\[
-P_i + \frac{\left( \frac{P_{i+1}}{P_i} \right)^\theta Y'_{i+1}}{1+i_{t+i}} = 0
\]

Since \( Y_{i+1} = Y', \) and \( p_{i+1} = P_{i+1} \) are obtained in the market equilibrium, the following equation is derived.

\[
\frac{K'_{i+1}}{L'_{i+1}} = \frac{P_{i+1}}{P_i} \frac{\left( \frac{P_{i+1}}{P_i} \right)^\theta Y'_{i+1}}{1+i_{t+i}} = 0
\]

(11)

In the same way, the following equation is derived from (10) when \( \partial \Theta_i / \partial K'_{i+1} = 0 \), and \( Y_{i+1} = Y', \) and \( p_{i+1} = P_{i+1} \):

\[
\frac{K'_{i+1}}{L'_{i+1}} = \frac{P_{i+1}}{P_i} \frac{\left( \frac{P_{i+1}}{P_i} \right)^\theta Y'_{i+1}}{1+i_{t+i}} = 0
\]

(12)

Since firms are symmetrical, \( \frac{Y'}{L'} = \frac{Y}{L} \) and \( \frac{K'}{L'} = \frac{K}{L} \) are obtained. Here, \( K \) indicates the volume of capital of the
whole economy, $y$ represents output-labor ratio $\left( \frac{Y}{K} \right)$, and $k$ represents capital-labor ratio $\left( \frac{K}{L} \right)$. From (7), (11), and (12), $\frac{Y}{L} = \frac{y}{L}$ and $\frac{K}{L} = \frac{k}{L}$, the following equations are derived:

$$y_{st} = (k_{st})^y$$

(13)

$$\left( k_t \right)^{\frac{1}{1+r}} = \frac{\beta(1 + r_{st})}{\alpha}$$

(14)

$$\left( k_t \right)^{\frac{1}{1-\alpha}} = \frac{\beta \sigma r}{1-\alpha}$$

(15)

If the volume of investment in the whole economy is represented by $I$, the following equation is derived from (9):

$$K_{st} = I_t$$

(16)

In this paper full-employment is assumed and in the two-period overlapping generations model only the savings of the younger generation is assumed to affect capital accumulation. Therefore, if the population of the generation bearing in period $t$ is represented by $L_t$, the following equation is derived from (16):

$$K_{st} = s_tL_t$$

(17)

If the constant ratio of population growth is represented by $n$, the following equation is derived from (5), (15), and (17):

$$k_t = \frac{(1-\alpha)(k_{st})^y}{\beta(2+\rho)(1+n)}$$

(18)

The phase diagram (Figure 1) is drawn by (18), and from the diagram it is clear that the capital-labor ratios converge into the equilibrium capital-labor ratio indicated by $k_i$ in Figure 1.

Here, we get the value of $k_i$. Since $k_i = k_{st} = k_{si}$, we have the following equation:

$$k_i = \frac{\left( 1 - \alpha \right)^{1/s}}{\beta(2+\rho)(1+n)}$$

(19)

Similarly, if the equilibrium output-labor ratio gained through dividing production volume by labor volume, which is the number of workers below retirement age, is represented by $y_i$, the following equation is derived from (13) and (19):

$$y_i = \left( 1 - \alpha \right)^{1/s} \frac{\beta(2+\rho)(1+n)}{\beta(2+\rho)(1+n)}$$

(20)

2. A model with elderly people working

In this section, we present our original overlapping generations model in which elderly people are engaged in work.

In our model, the generation bearing at $t$ period is assumed to adopt the following behaviors.
They divide their wages \( (\omega'_{i,t}) \) when they are younger (period \( t \)) into consumption \( (c'_{i}) \) and savings \( (s'_{i}) \).

When they are older (period \( t+1 \)), they spend their savings including interest \( \left( 1 + r_{i,t} \right) s'_{i} \) and their wages \( (\omega'_{i,t+1}) \).

From the behaviors mentioned above, the following utility function and budget constraints are drawn:

\[
U = \log(c'_{i}) + \log(\frac{c'_{i,t+1}}{1 + \rho})
\]

\[
c'_{i} + s'_{i} = \omega_{i}
\]

\[
c'_{i,t+1} = (1 + r_{i,t})s'_{i} + \omega'_{i,t+1}
\]

The following equation is derived from (21) to (23):

\[
U = \log(\omega_{i} - s'_{i}) + \log \left( \frac{1 + r_{i,t} s'_{i} + \omega'_{i,t+1}}{1 + \rho} \right)
\]

Then, in order to find the level of optimal savings of a worker who bears in period \( t \) \((s'_{i})\), let \( \partial U / \partial s'_{i} = 0 \). Then, the next equation is derived:

\[
s'_{i} = \frac{\omega_{i}}{2 + \rho} - \frac{\omega'_{i,t+1}(1 + \frac{\rho}{\rho})}{(1 + r_{i,t})(2 + \rho)}
\]

In our original model in which elderly people working is assumed, the production function of firm \( j \) is represented as follows:

\[
Y_{j} = (K_{j}^{1})^{\gamma} \left( L_{j}^{\gamma} + \gamma L_{j}^{1-\gamma} \right)^{1-\gamma} \quad \gamma < 1
\]

\( Y'_{j} \): number of workers who have born in period \( t-1 \) and work in period \( t \). (number of elderly workers)

\( L'_{j} \): number of workers who bear and work in period \( t \). (number of young workers)

\( \gamma \) in (26) is a parameter that indicates the productivity reduction caused by aging. Let \( L'_{j} \) stand for the quantity of labor employed by firm \( j \) during period \( t \), the following equation is obtained:

\[
L'_{j} = L'_{j}^{1} + L'_{j}^{1-1}
\]

In our original model, firm \( j \) tries to maximize its objective function \( \Theta_{2} \) shown in (27) at the time of investment decision.

\[
\Theta_{2} = P_{j}^{1} + \frac{\left( p'_{i,t+1} Y_{j,t+1} - w'_{i,t+1} L_{j}^{1} - \omega'_{i,t+1} L_{j}^{1} \right)}{1 + i_{t+1}}
\]

From (7) to (9) and (26), (27), the objective function \( \Theta_{2} \) is as follows:

\[
\Theta_{2} = \frac{\left( p'_{i,t+1} y_{j,t+1} - \frac{w'_{i,t+1} L_{j}^{1}}{P_{j}^{1}} - \frac{\omega'_{i,t+1} L_{j}^{1}}{P_{j}^{1}} \right)}{1 + \frac{i_{t+1}}{\rho}}
\]

In equation (28), \( w'_{i,t+1} \) denotes the nominal wage rate paid at period \( t+1 \) for the generation bearing in period \( t \), and \( w'_{i,t+1} \) also denotes the nominal wage rate paid at period \( t+1 \) for the generation bearing in period \( t+1 \).

In order to solve the profit-maximization problem, let \( \partial \Theta_{2} / \partial K_{j,t}^{1} = 0 \). Then the following equation is derived.

\[
\left( \frac{p'_{i,t+1}}{P_{j}^{1}} \right)^{\gamma} \left( \frac{w'_{i,t+1}}{P_{j}^{1}} \right)^{1-\gamma} = 0
\]

Since \( Y_{j,t+1} = \frac{Y_{j,t}}{\rho} \), and \( p'_{i,t+1} = P_{j}^{1} \) are obtained in the market equilibrium, the following equation is derived.

\[
\left( \frac{K_{j,t+1}}{L_{j,t+1}} \right)^{\gamma} = \frac{\beta(1 + r_{i,t})}{\alpha}
\]

In the same way, the following equation is derived from (27) when \( \partial \Theta_{2} / \partial L_{j}^{1} = 0 \), \( Y_{j,t+1} = Y_{j,t} \), and \( p'_{i,t+1} = P_{j}^{1} \):

\[
\left( \frac{K_{j,t+1}}{L_{j,t+1}^{1} + \gamma L_{j,t+1}^{1-1}} \right)^{\gamma} = \frac{\beta(1 + r_{i,t})}{\alpha}
\]

Moreover, in the same way, the following equation is derived from (10) when \( \partial \Theta_{2} / \partial L_{j}^{1} = 0 \), \( Y_{j,t+1} = Y_{j,t} \), and \( p'_{i,t+1} = P_{j}^{1} \), the following is derived:

\[
\left( \frac{K_{j,t+1}}{L_{j,t+1}^{1} + \gamma L_{j,t+1}^{1-1}} \right)^{\gamma} = \frac{\beta(1 + r_{i,t})}{\alpha}
\]

Here, we present the following equation, which is useful for the discussion that follows.

\[
L'_{j} + \gamma L'_{j} = \left( \frac{1 + n + \gamma}{2 + n} \right) L_{j} + \frac{A}{2 + n}
\]

Since firms are symmetrical, \( \frac{Y'_{j}}{L'_{j}} = \frac{Y}{L} \) and \( \frac{K'_{j}}{L'_{j}} = \frac{K}{L} \) are obtained. Therefore, from (29) to (32), we have

\[
1 + r_{i,t} = \frac{a(k_{i,t})}{\beta A^{\gamma}}
\]
\[ a_i' = \frac{(1 - \alpha)k_i}{A_i'^2} \]  
\[ a_i'^{-1} = \frac{\alpha(1 - \alpha)k_i}{A_i'} \]  

As already described, since only the savings of the younger generation is assumed to affect capital accumulation in our two-period overlapping generation model, the equation of capital stock of the next period (36) is similar to the equation (17) in section 2.

\[ K_{i+1} = s_i'L_i' \]  

Therefore, from (25) and (33) to (36), the following equation is derived:

\[ k_{i+1} = \frac{B(k_i)^{\gamma}}{1 - C} \]

\[ B = \frac{1 - \alpha}{\beta(2 + \rho(2 + n))^{A_i'}} \quad C = \frac{(1 + \rho)\gamma(1 - \alpha)}{(2 + \rho)\alpha(2 + n)} \]  

Since the power attached to \( k \) in (18) and (37) is the same, the phase diagram by (37) is analogous to the phase diagram shown in Section 2. Therefore, capital-labor ratios converge into equilibrium capital-labor ratio \( k \). Since \( k \) is calculated as \( k_{i+1} = k_i = k_{i+1} = k_{i+2} \), we have the following equation:

\[ k_i = \left[ \frac{\alpha(1 - \alpha)\gamma}{\beta(2 + \rho)(1 + n + \gamma)\alpha + (1 + \rho)(1 - \alpha)\gamma} \right]^{\frac{1}{1 - \alpha}} \]  

The denominator of the capital-labor ratio \( k \) represents the total number of both young and elderly workers. In contrast, in the capital-labor ratio \( k_i \) presented in Section 2, the denominator represents the number of young workers only. In order to compare the typical overlapping generations model and our original model, the capital-labor ratio whose denominator is limited to the number of young workers only must be obtained in the framework of our original model. When this capital-labor ratio is denoted by \( k_i' \), the following equation is derived from (32) and (38):

\[ k_{i+1} = \left[ \frac{\alpha(1 - \alpha)\gamma}{\beta(2 + \rho)(1 + n + \gamma)\alpha + (1 + \rho)(1 - \alpha)\gamma} \right]^{\frac{1}{1 - \alpha}} \]

Then, from (26), the following equation is derived:

\[ Y = \frac{1 + n + \gamma}{\gamma} k_i'^{1 - \alpha} \]  

Therefore, if \( y_i' \) denotes the ratio gained through dividing the quantity of production by the number of young workers, the following equation is derived from (39) and (40):

\[ y_i' = \left[ \frac{\alpha(1 - \alpha)\gamma}{\beta(2 + \rho)(1 + n + \gamma)\alpha + (1 + \rho)(1 - \alpha)\gamma} \right]^{\frac{1}{1 - \alpha}} \]  

\[ E = \frac{(1 + n + \gamma)^{1 - \alpha}}{1 + n} \]  

3. Elderly people working, Capital accumulation, and Production level

In this section, we analyze the influence of elderly people working on capital accumulation and production level based on the analyses in Sections 2 and 3. First, we investigate the influence of elderly people working on capital accumulation. We compare two capital-labor ratios \( k_i \) and \( k_i' \) obtained in Section 2 and Section 3 respectively. The reason we compare \( k_i \) and \( k_i' \) instead of \( k_i \) and \( k_i'' \) is that \( k_i \) and \( k_i' \) have the same denominators showing the number of workers (the number of young workers). This enables us to compare their capital volumes, which are the numerators. This comparison clarifies which economy model promotes greater capital accumulation, the typical overlapping generations model or our original overlapping generations model with elderly people working.

Since the power attached to the curly braces in equation (19) for \( k_i \) is the same as that in equation (39) for \( k_i' \), the comparison of \( k_i \) and \( k_i' \) is the same as the comparison of the fractions in the curly braces in (19) and (39). If we represent the fractions in (19) and (39) by \( v_i \) and \( v_i' \) respectively, \( v_i > v_i' \) is obtained as shown by the following calculation:

\[ v_i - v_i' = \frac{1 - \alpha}{\beta(2 + \rho)(1 + n)} \left[ \frac{\alpha(1 - \alpha)}{\beta(2 + \rho)(1 + n + \gamma)\alpha + (1 + \rho)(1 - \alpha)\gamma} \right]^{\frac{1}{1 - \alpha}} = \frac{\alpha(1 - \alpha)(2 + \rho)(1 + n + \gamma) + (1 + \rho)(1 - \alpha)\gamma}{(2 + \rho)(1 + n)(2 + \rho)(1 + n + \gamma)\alpha + (1 + \rho)(1 - \alpha)\gamma} \]

\[ \cdot \frac{(1 + n + \gamma)(1 + n + \gamma)^{1 - \alpha} - (1 + n + \gamma)^{1 - \alpha} - (1 + \rho)(1 - \alpha)^{1 - \alpha}}{0} > 0 \]

Therefore, the capital-labor ratio in the case in which elderly people are engaged in work is smaller than the ratio in the case in which elderly people are not engaged
in work. This proves that an economy with elderly people working has a smaller volume of capital in each period than an economy without elderly people working. After all, elderly people working does not promote capital accumulation as much as in the case that elderly people are not working.

The reason that elderly people working relatively hampers capital accumulation is that the younger generation, agent of savings, relatively decreases their savings because they know they can continue to work when they are older. It is clearly understood by comparing equation (25), which shows the level of optimal savings in the case in which elderly people engaged in work, and equation (5), which shows the level of optimal savings in the case in which elderly people are not engaged in work.

Next, we examine the influence of elderly people working on the level of production. We compare output-labor ratios \( y_i \) and \( y_i' \) obtained in Sections 2 and 3 respectively. The reason we compare \( y_i \) and \( y_i' \) is that \( y_i \) and \( y_i' \) have the same denominators showing the number of workers.

Since the power attached to the curly braces in equation (20) for \( y_i \) is the same as that in equation (41) for \( y_i' \), the comparison of \( y_i \) and \( y_i' \) is the same as the comparison of the fractions in (20) and (41). If we represent the fractions in (20) and (41) by \( u_i \) and \( u_i' \) respectively, the following calculations can be made:

\[
u_i - u_i' = \frac{1 - \alpha}{\beta(2 + \rho)(1 + n)} \frac{\alpha}{\beta(2 + \rho)(1 + n + \gamma) + (1 - \alpha) y_i' - \alpha y_i' - \alpha y_i'} - \alpha y_i - \alpha y_i' - \alpha y_i'
\]

\[
= \frac{\alpha(1 - \alpha)[(1 + n + \gamma)^{\frac{1 - \pi}{\gamma}} - (1 + n + \gamma)^{\frac{1 - \alpha}{\alpha}}]}{(1 + n + \gamma)^{\frac{1 - \pi}{\gamma}} - (1 + n + \gamma)^{\frac{1 - \alpha}{\alpha}}}
\]

In order to examine the sign of \( (u_i' - u_i) \), let the numerator of the last fraction in (42) be \( X \). Then we get the following equation:

\[
X = \alpha(1 - \alpha)(2 + \rho)Z + (1 + \rho)(1 - \alpha)^{\frac{1 - \pi}{\gamma}}
\]

Where \( Z \) shows the following

\[
Z = (1 + n + \gamma) - (1 + n)^{\frac{1 - \pi}{\gamma}} - (1 + n + \gamma)^{\frac{1 - \alpha}{\alpha}}
\]

In (43), if \( Z \) is positive, \( X \) is also positive. Therefore, in (42), if \( X \) is positive, \( (u_i' - u_i) \) is also positive.

Notice here in (44), the sign of \( Z \) is related to the size of \( \alpha \).

If \( \alpha \) is bigger than 1/2, \( Z \) is positive from the following calculations.

\[
1 + n + \gamma = (1 + n + \gamma)^{\frac{1 - \pi}{\gamma}} - (1 + n + \gamma)^{\frac{1 - \alpha}{\alpha}} > (1 + n)^{\frac{1 - \pi}{\gamma}} - (1 + n + \gamma)^{\frac{1 - \alpha}{\alpha}}
\]

Then, in this situation, \( X \) and \( (u_i' - u_i) \) are positive.

In contrast, if \( \alpha \) is smaller than 1/2, \( Z \) is negative from the following calculations.

\[
(1 + n + \gamma)^{\frac{1 - \pi}{\gamma}} < 1 + n + \gamma \quad (1 + n)^{\frac{1 - \pi}{\gamma}} > \frac{1}{(1 + n)^{\frac{1 - \alpha}{\alpha}} - 1} > 0
\]

Therefore, in this situation, there are some cases where \( X \) and \( (u_i' - u_i) \) are negative. Moreover, through further calculation the following inequality can be derived:

\[
\frac{\partial X}{\partial \alpha} < 0
\]

For example, in the case of \( \alpha = \frac{1}{4}, n = 1, \gamma = 1, X \) has the following value, which is negative.

\[
X = -\frac{(8 + 3\rho)}{16}
\]

In the analyses above, it is necessary to note that \( y_i \) and \( y_i' \) can be derived in the case of \( \alpha > 1/2 \). This indicates that the production level when elderly people are not engaged in work is higher than the level when elderly people continue to work.

This conclusion is surprising, because the production level of an economy having a relatively large number of workers is lower than that of an economy having a relatively small number of workers.

This conclusion results from the fact that elderly people working has a negative influence on capital accumulation. In the case where elderly people are engaged in work, the capital volume in each period relatively decreases, because the savings of young workers remains relatively low.

There are some cases in which relative capital shortage lowers the production level.

However, in our model, although elderly people working has a negative influence on capital accumulation, it does not always have a negative influence on production level. This point makes our model different from the model of Futagami and Nakajima (2001) as mentioned in the first section. It is related to the fact that the production-capital ratio \( \left( \frac{Y_i}{K_i} \right) \) changes in our model. In contrast, in the model of Futagami and Nakajima (2001) the production-capital ratio remains constant. Unlike in Futagami and Nakajima (2001), in our model production-labor ratio \( y_i' \) is represented.
as follows:

\[ y_2 = \left( \frac{Y}{K} \right) _2 \left( \frac{K}{L} \right) _2 \]  

(45)

Equation (45) indicates that, even if elderly people working decreases \( \frac{K}{L} \), \( \frac{Y}{L} \), can increase on the condition that \( \frac{Y}{K} \) increases at the same time.

4. Conclusion

In this paper, we compare the typical overlapping generations model where the older generation (elderly people) are not engaged in work and our original overlapping generations model where the older generation are engaged in work. By this comparison, we examine what influence elderly people working exerts on capital accumulation and production level.

We arrived at two conclusions.

(1) An economy with elderly people working has less capital volume in each period than an economy without elderly people working.

(2) An economy with a relatively large number of workers because of elderly people working may have a lower production level than an economy with a relatively small number of workers because of elderly people not working.

The first conclusion is based on the following reason: if elderly people are engaged in work, the younger generation will relatively decrease their savings in their youth, because they plan to continue working and earning when they are older. The second conclusion, which relates to the first, is particularly notable. Since an economy with elderly people working causes capital shortage compared with an economy without elderly people working, there are some cases in which the economy with elderly people working relatively reduces production level.

\[ \text{Notes} \]

1 The features of this model are already well known. For example, see Wakita (1998) pp.113-114.

2 This individual demand function is based on Blanchard and Fisher (1989) and Blanchard and Kiyotaki (1987).

3 If the output-capital ratio without elderly people working stands for \( \left( \frac{Y}{K} \right) _1 \) and the ratio with elderly people working stands for \( \left( \frac{Y}{K} \right) _2 \), the following equations are derived:

\[ \left( \frac{Y}{K} \right) _1 = \left( \frac{1}{k} \right) ^{1-\alpha} \]  

\[ \left( \frac{Y}{K} \right) _2 = \left( \frac{2 + n}{(1 + n)k} \right) ^{1-\alpha} \]

Because of \( k_2 > k_1 \), the output-capital ratio of an economy with elderly people working may be higher than that of an economy without elderly people working, in our model.

References


